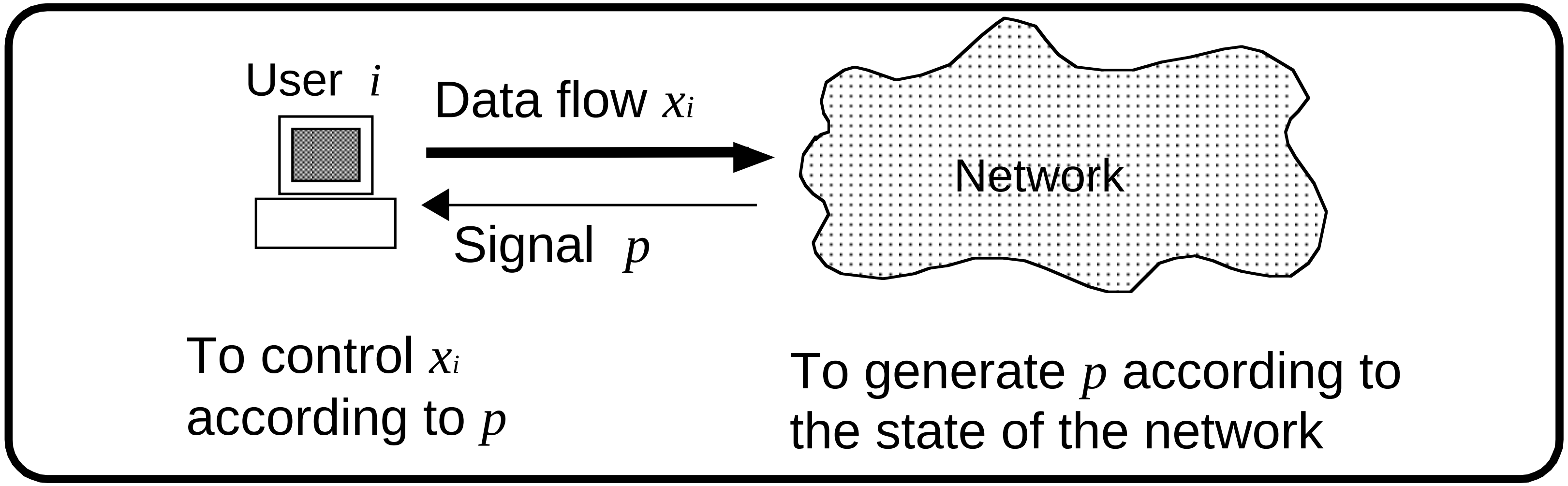
Marginal Waiting Cost in Optimization Based Flow Control

Toshihisa Ozawa
Department of Business Administration
Komazawa University, Japan
toshi@komazawa-u.ac.jp

Flow control (congestion control)



Example:

TCP flow control & random early detection (RED)

p: Marking in an ACK packet

x_i corresponds to congestion window size.

ABR flow control

p: Congestion indication (CI) and/or explicit rate (ER) in an RM cell

x_i corresponds to allowed cell rate (ACR)

Flow control as an optimization problem

To construct a flow control mechanism

--> To determine the following according to **a policy**

- How to generate p
- How to control x_i according to p

Optimization based flow control

- Policy

  --> To minimize the total cost, which is congestion cost minus user utilities

- Mechanism to generate p and to control x_i

  --> Solution for the optimization problem corresponding to the policy

Related works

- F. P. Kelly et al., Rate control for communication networks: shadow price, proportional fairness and stability, Journal of the Operational Research Society, 49 (1998).
- S. J. Golestani et al., A class of end-to-end congestion control algorithms, Sixth International Conference on Network Protocols (1998).
- S. H. Low et al., Optimization flow control, I: basic algorithm and convergence, IEEE/ACM Transactions on Networking (1999).

Kelly et al. (1998)

- Policy (single bottleneck case)

$$\max_{\mathbf{x}} J(\mathbf{x}) \equiv \sum_i U_i(x_i) - R\left(\sum_i x_i\right)$$

$U_i(\cdot)$: Utility function of user i, strictly increasing, concave, differentiable

$R(\cdot)$: Congestion cost of the bottleneck link, differentiable

$p(y) = \mathrm{d}/\mathrm{d}y\, R(y)$: shadow price, positive, strictly increasing

- Mechanism to control x_i (willingness to pay type control)

$$d/dx\ x_i(t) = \kappa_i\left(w_i(t) - x_i(t)p\left(\sum_j x_j(t)\right)\right)$$

$$w_i(t) = x_i(t)U'(x_i(t))$$

κ_i: control parameter

Our approach

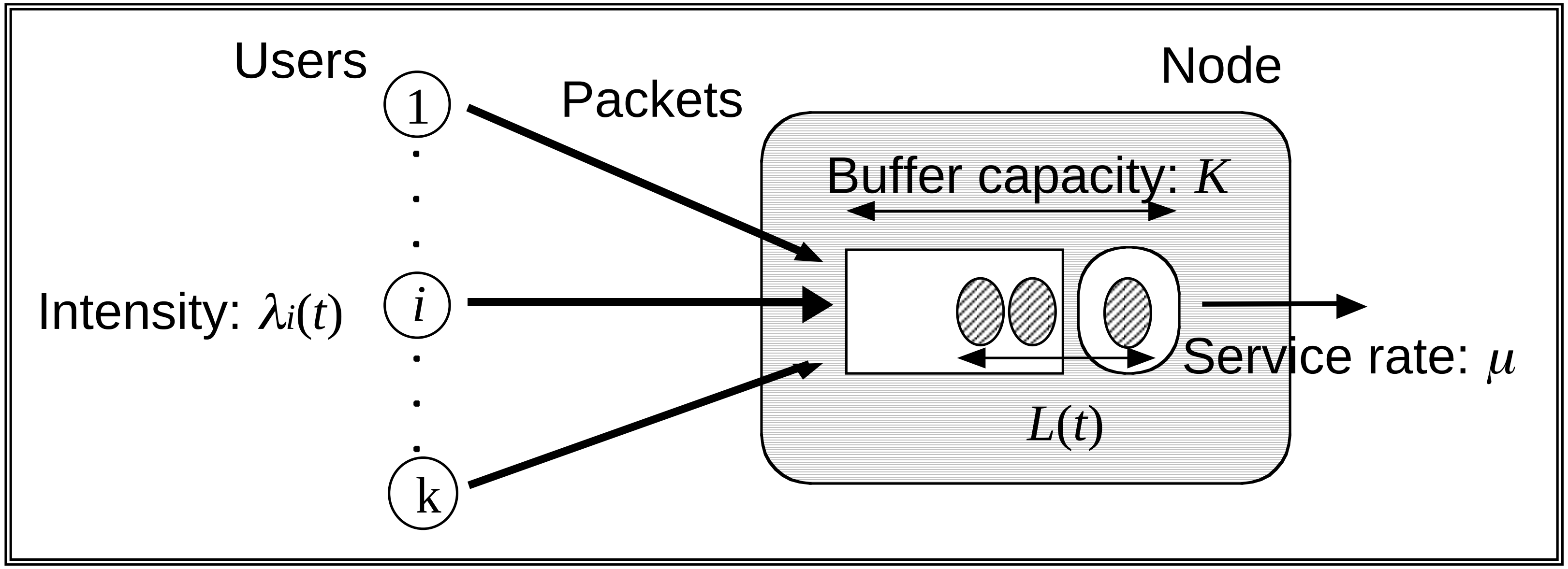
- To consider **accumulated cost**, for example,

$$\text{Minimize} \int \left(\sum_i U_i(x_i(t)) - R\left(\sum_i x_i(t) \right) \right) dt$$

- To consider **stochastic models**, which are queueing models

Model

Single bottleneck model: single node and multiple users



- Poisson arrival processes:
  intensity vector $\lambda(t) = (\lambda_i(t))$ <-- **control subject**
- Exponential distributed services:
  service rate μ (the same for all the users)
- K: Buffer size, including service facility
- $L(t)$: the number of packets in the system at time t
- $A_i(t)$: the number of user i's packets arriving in $[0, t]$

User utility and congestion cost

User utility is represented as a function of throughput.

$U_i(1(L(t) < K)\ \lambda_i(t))$: **Instantaneous utility** of user i at time t

$U_i(\cdot)$: a function, non-negative, increasing, differentiable, strictly concave

$$\mathrm{E}\left[\frac{1}{\mathrm{t}}\int_0^t 1(L(t-) < K)\, dA_i(t)\right] = \mathrm{E}\left[\frac{1}{\mathrm{t}}\int_0^t 1(L(t) < K)\lambda_i(t)dt\right]$$

1(·): indicator function

--> From this formula, $1(L(t) < K)\ \lambda_i(t)$ can be considered as **the instantaneous throughput** of user i

Congestion cost approximately represents delay and loss.

$R(L(t))\ \lambda_i(t)$: **Instantaneous congestion cost** for user i a time t

$R(\cdot)$: a function, non-negative, increasing (the same for all the users)

Note: $R(K)\ \lambda_i(t)$ represents the instantaneous cost of loss.

Formulation 1: Expected discounted cost

To minimize the expectation of the discounted total user cost

$\mathbf{u}(t) = (u_i(t))$: Control parameters, i.e., $\lambda_i(t)$ <-- $u_i(t)$

$B_i = [b_{i,0}, b_{i,1}]$: Range of $u_i(t)$, $\mathbf{B} = B_1 \times ... \times B_k$

$U_{i,\max} = U_i(b_{i,1})\, b_{i,1}$

$\alpha > 0$: Exponential discount factor

Instantaneous cost for user i

$$C_i(L(t), \lambda_i(t)) \equiv \left\{ U_{i,\max} - U_i\left(1(L(t) < K)\lambda_i(t)\right)\right\} + R(L(t))\lambda_i(t)$$

$$C(L(t), \lambda(t)) \equiv \sum_{i=1}^{k} C_i(L(t), \lambda_i(t))$$

Objective function

$$J_\alpha(\mathbf{u}, l_0) \equiv \lim_{T \to \infty} \mathrm{E}_{\mathbf{u}}\left[\int_0^T e^{-\alpha t} C(L(t), \mathbf{u}(t))dt \mid L(0) = l_0\right]$$

Policy (criterion)

Minimize $J_\alpha(\mathbf{u}, l_0)$

Result 1: Expected discounted cost

Problem 1: P1

$$J_\alpha^*(l_0) \equiv \inf_{\mathbf{u}} J_\alpha(\mathbf{u}, l_0)$$

Result

Assume that there exist the function $V_\alpha(\cdot)$ that satisfies

(1) $$V_\alpha(l) \equiv \inf_{\mathbf{u}} J_\alpha(\mathbf{u}, l),$$

and the function $v^*_\alpha(\cdot)$ that satisfies

(2) $$\mathbf{v}_\alpha^*(l) \equiv \arg\min_{\mathbf{v}\in\mathbf{B}} \left\{ \sum_{i=1}^{k} v_i 1(l < K)\left[V_\alpha(l+1) - V_\alpha(l)\right] + C(l, \mathbf{v}) \right\}.$$

Then, the optimal control $\mathbf{u}^*_\alpha$ is given by

(3) $$\mathbf{u}_\alpha^*(t) = \mathbf{v}_\alpha^*(L(t)).$$

This result is directly derived from Theorem VTT-T1 in Point Processes and Queues by Bremaud.

Formulation 2: Expected average cost

To minimize the expectation of the average total user cost

Objective function

$$J(\mathbf{u}, l_0) \equiv \lim_{T \to \infty} \mathrm{E}_{\mathbf{u}} \left[\frac{1}{T} \int_0^T C(L(t), \mathbf{u}(t)) dt \mid L(0) = l_0 \right]$$

Policy (criterion)

Minimuze $J(\mathbf{u}, l_0)$

Result 2: Expected average cost

Problem 2: P2

$$J^*(l_0) \equiv \inf_u J(u, l_0)$$

Result

Assume that there exist the function $V\alpha(\cdot)$ that satisfies

(1) $$V_\alpha(l) \equiv \inf_u J_\alpha(\mathbf{u}, l),$$

and the function $G(\cdot)$ that is a certain limit of $V\alpha(.)$, i.e,

(4) $$G(l) \equiv \lim_{n\to\infty}\left(V_{\alpha_n}(l+1) - V_{\alpha_n}(l)\right), \text{ where } \alpha_n \to 0 \text{ as } n \to \infty.$$

Furthermore, assume that there exists the function $\mathbf{v}^*(\cdot)$ that satisfies

(5) $$\mathbf{v}^*(l) \equiv \arg\min_{\mathbf{v}\in\mathbf{B}}\left\{\sum_{i=1}^{k} v_i 1(l < K) G(l) + C(l, \mathbf{v})\right\}.$$

Then, the optimal control **u*** is given by

(6) $$\mathbf{u}^*(t) = \mathbf{v}^*(L(t))$$

Discussion

Functions $V\alpha(\cdot)$ and $G(\cdot)$

$V\alpha(\cdot)$ is the expected total cost for the future.

$$V_\alpha(l) \equiv \inf_{\mathbf{u}} J_\alpha(\mathbf{u}, l)$$

G(·) can be considered as the expected cost of one packet for the future.

$$G(l) \equiv \lim_{n \to \infty} \left(V_{\alpha_n}(l+1) - V_{\alpha_n}(l) \right)$$

Flow control (the case of expected average cost)

From equation (5), $\mathbf{v}^*(\cdot)$ satisfies

(7) $$U_i'(v_i^*(l)) \equiv G(l) + R(l), \quad l < K.$$

Therefore, signal $p(\cdot)$ can be define as

$$p(l) \equiv G(l) + R(l),$$

and the packet arrival intensities are given as the solution of

(P3) $$\inf_{v_i} \left\{ v_i p(L(t)) - U_i(v_i) \right\}.$$

Note: $p(\cdot)$ corresponds to a shadow price and (P3) to Kelly's willingness to pay type control.

Further study

- How to generate signals using only data obtained by the network

- Analysis of a model that includes delay of signal